REVISITING STOCK MARKET ASSUMPTIONS. APPLICATION FOR ROMANIAN STOCK EXCHANGE

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Abstract. *The last century was marked by a prodigious development of modeling phenomena of capital markets. During the basic process, a number of assumptions that were made proved to have more or less validity in tests confronting or conforming to reality. In this paper we propose to question some of those assumptions in the case of the capital market of Romania.*

Keywords: *Efficient Market Hypothesis, stable distribution, predictability.*

1. INTRODUCTION

Understanding the phenomena that take place within the capital markets or those connected to it, requires the use of quantitative methods and techniques.

In particular, the statistical methodology provides efficient tools for the analysis of the processes taking place in close connection with capital markets activity.

After centuries of studies in which statistics was used to investigate a wide variety of phenomena in economic or social areas, medical sciences etc., a useful conclusion can be drawn for the understanding of these classes of events: unlike sciences terminology, such as logic, statistics does not operate with concepts of true or false. A statistical conclusion will almost never be true or false, but only probable or less probable, the accuracy test of this conclusion being comparison with reality.

This requires a special relationship with reality: to understand a phenomen of this nature, usually we seek to obtain a law, i.e. a general rule of governing the intrinsic life of that phenomenon; and we discover this by building models of reality, being more or less abstract ones, trying to surprise the essence of the event.

In general, a model is a simplified representation of reality, taken in order to understand its essential aspects.

In economics, a model is a theoretical construct explaining the economic process through a set of variables and some qualitative and quantitative relationships between them.

The model simplifies, being a key to understanding reality, not at a true reflection of it.

The way in which we use models to represent and have a deep knowledge of reality allows us to form an analogy with Plato's theory of ideas.

According to this theory, there is a perfect world, the world of ideas, of forms, which is the true reality, the sensory world being a pale reflection of the world ideas. Thus, having an idea of a perfectly round circle cannot be false as is relating to and describing an aspect of perfection. However, this idea does not prove the existence of anything, but merely demonstrates that it is possible for an imperfect being to hold a concept of perfection. If it is possible to hold an idea of a perfect circle, but still be imperfect, surely it is possible to hold a concept of a perfect being whilst being imperfect.

This happens generally with geometric models, which operates with perfect concepts and notions that are not to be found in inherent reality.

In general, a model is in intrinsic connection with the reality that it describes; starting from the actual data of reality, we are building models, schemes of understanding of reality. These models can be studied independently, at the abstract level (which it makes it particularly applicable to modern mathematics), have a life of their own, in the ideal world of models, but ultimately there is a return to real world models with valid conclusions from the reality's level that gave rise to the model.

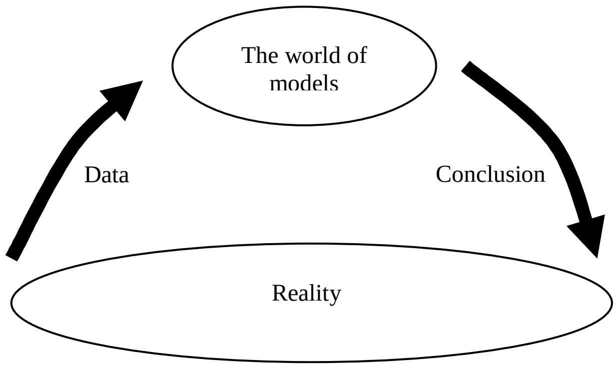

Fig. 1. Scheme of understanding reality through a model

We distinguish between two classes of models: deterministic models and probabilistic models.

Deterministic models are models in which parameters and variables are not subject to some random fluctuations. An example is the model associated with Newtonian mechanics. According to the second principle of mechanics, the force is proportional to the body's mass in motion and it's accceleration: $\vec{F} = m\vec{a}$. Then whenever we know the values of mass and acceleration of a body in motion, we know with certainty the amount of force developed by it. In this case the only errors that can interfere are errors of measurement.

Probabilistic models are models that take into account the random component. In our attempt to include in a mathematical model the surrounding reality, a special requirement is the prediction of the future states of reality with the help of the build model. Since the use of deterministic models to capture essential aspects of reality seems to be an approach inevitably doomed to failure[1], a reasonable solution would be to use stochastic models, in which the random factor is given a proper role.

Modeling capital markets has a long history, which can not be separated from the historical development of modern methods and techniques of quantitative investigation.

As it will be seen from this work, modeling capital market is in a close relationship with the hypothesis of capital market efficiency, a concept also correlated with the rationality of the participants's behavior in the market's activities.

In the following analysis we make an overview of the main authors who have approached this piece of reality in their work, but also the most important results that have marked in a definitive way the understanding of phenomena that occur in the capital markets.

In the year 1565, the famous Renaissance mathematician Girolamo Cardano published the book *Liber de ludo aleae (The book of gambling)*, in which he proposes the concept of equality of chances, which can be found in modern literature of game theory under the name of the correct game (fair game): „the fundamental principle in all gambling is the equality of chances, whether it is about opponents, money, etc.".

In the year 1828, the Scottish botanist Robert Brown observed in an experiment that pollen particles in suspension have a random oscillatory movement, which rapidly change the trajectory. This observation leads to the concept of the Brownian motion.

In 1863 a French broker, Jules Regnault noted a fundamental property of Brownian motion: the variability (measured by standard deviation) of Brownian motion is proportional to the square root of time.

It is already the moment when it begins to crystallize the main notions and concepts that will mark the effort of modeling financial phenomena.

Thus, British physicist Rayleigh discovered in 1880 the processes of random walk (walking at random) during his studies on sound waves.

In the year 1888, logician and philosopher John Venn formulated a coherent concept of random walk and Brownian motion.

The crucial point in modeling financial market phenomena came in the year 1900, when the young French mathematician Louis Bachelier published his doctoral thesis *Théorie de la speculation*[2]. Using statistical methods, he deduced that the mathematical expectation of the speculator is zero; also, it formalized the Brownian motion, being calculated the probability that a certain return to be achieved in a given period of time.

In 1905, the statistician Karl Pearson introduced the term of *random walk* in his article in the journal *Nature*[3] where he define the random walk proccess: „in an open space, the most likely place where to find a drunken man, who can not stand up, is somewhere in the neighborhood of its initial position".

That same year, independent of previous research by Bachelier, Einstein developed the equations that are describing Brownian motion.

Much later, in the 70's, appears the article that will definitely mark the theory of financial markets: *Efficient Capital Markets: A Review of Theory and Empirical Work*, written by Fama. This paper provides a summary of previous research on the issue of predictability in equity markets and it provides clearly formalism for notions like fair game and random walk.

Also, it is given the classic definition of efficient markets: a market is efficient if stock prices always fully reflects all available information[4].

There is made the distinction between three forms of efficiency: strong, semi-strong and weak form efficiency. To discuss capital market efficiency hypothesis to be considered common (*joint hypothesis*): in addition to studying how a stock price fully reflects available information should be considered an investor's attitude towards risk. As will be shown later Campbell, Lo and MacKinlay (1997) [5] „any test of efficiency is based on the assumption that there is an equilibrium model that defines the normal gains. If efficiency is rejected, this may mean that the market is truly inefficient or may be a sign that an inappropriate model of market equilibrium was chosen".

It is interesting to note that these papers, as many others, are based on several assumptions that are more or more or less explicitly formulated: hypothesis of rational behavior of investors, independence of economic agents, hypothesis of existing a normal distribution of returns, assuming the existence of market equilibrium.It may be questioned whether these assumptions are originating from economic reality or all of the past decade of research has modified the reality according to the theory. In the following we discuss in a critical way some of these assumptions.

2. NORMAL DISTRIBUTION ASSUMPTION

In 1915, Wesley Mitchell argued[6] that the distribution of financial asset price changes is „too stretched" to come from a normal distribution.

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[1] See in this respect also the aroused controversy in the XVIIth and XIXth centuries on the deterministic models of the Universe built by Laplace which is assigned the following phrase:" No place for God in the world build by me" (i.e., no error would have belong in such a purely, deterministic Universe!).

[2] Bachelier, L.,1900, "Théorie de la speculation", *AnnalesScientifique de l'E.N.S.*,3^{e}serie, tome 17,pp.21-86,http://www.numadam.org/item ?id=ASENS_1900_3_17_21_0

[3] Pearson, K. , 1905, "The Problem of Random Walk", *Nature*, No.1865, Vol. **72**, August.

[4] A market in which prices always "fully reflect" available information is called "efficient".

[5] Campbell, J.Y., LO, W., MacKinlay, C., 1997, *The Econometrics of Financial Markets*, Princeton University, pp. 24.

[6] Mitchell, W. C., 1915 and 1921, "The Making and Using of Index Numbers," Introduction to *Index Numbers and Wholesale Prices in the United States and Foreign Countries*, published in 1915 as Bulletin No. 173 of the U.S. Bureau of Labor Statistics, reprinted in 1921 as Bulletin No. 284.

In 1923 Keynes[7] emits the hypothesize that investors in financial markets obtain profits not due their ability to predict better than the overall market's future price developments, but because of an appetite for risk, an idea that is consistent with the efficient market hypothesis.

In 1926 the French mathematician Maurice Olivier[8] provided a clear demonstration that the return distribution on the capital market is a leptokurtic distribution, which deviates from the normal distribution curve, being more elongated than that.

In an article from 1960[9], Larson shows that the return distribution is very closed to the normal for the 80% of observations in the middle of the original distribution, but there are a large number of extreme values that creates a departure from normality.

One of the researchers who decisively influenced financial modeling, Benoit Mandelbrot, rediscovers in an article in 1962 (but published a year later[10]) the ideas of Louis Bachelier and proposes the so-called stable distributions, of Pareto-Levy type, for price behavior of a financial asset, which explains better than normal distribution the occurrence of extreme values.

A confirmation that stock's return (calculated as the difference of logarithms of prices) follows a stable Pareto distribution is obtained by Fama in 1963[11]: thus, the logarithm of the characteristic function of a stable Pareto distribution has the form:

$$\ln\varphi(t)=\ln\int_{-\infty}^{+\infty}\exp(uit)dP(\tilde{u}<u)=i\delta t-\gamma|t|^{\alpha}\left[1+i\beta\frac{t}{|t|}\tan\left(\frac{\alpha\pi}{2}\right)\right].$$

Mandelbrot demonstrated that the parameter α is controlling the length of „tails" of such a distribution and takes values in the range $[0,2]$. In particular, if $\alpha=2$, the stable Pareto distribution becomes a normal distribution.

Using daily differences of logarithms share prices of 30 companies from the Dow Jones Industrial Average, Fama estimates the α parameter and obtains in all the cases values smaller than 2, an important argument in the favour of the fact that the return is not following a normal distribution, but a stable Pareto one.

Fama's calculation on the distribution of returns indicates that they are following a stable Pareto distribution, with coefficient $\alpha<2$, a strong argument against the hypothesis of normal distribution.

An important consequence arising from this conclusion: since the Pareto distribution with $\alpha<2$ has the second-order infinite, therefore infinite dispersion, using classical methods of estimation, such as ordinary least squares method, becomes unnecessary. Fama suggests using mean absolute deviation[12] instead of variance as a measure of variation.

More recents works (Rachev, etc.) rediscover the theory of stable distributions in financial modeling theory and shows that there are much better approaches than classical distributions. The fact that the observed distribution of the returns is heavy-tailed can not be explained through a normal distribution. Further, the frequency of extreme events such as the financial crisis is much bigger than it actually allowing for Gaussian distribution.

3. EFFICIENT MARKET HYPOTHESIS

Efficient market idea, as it is understood in modern literature, has its origins from Bachelier Cowles and Samuelson's works. In 1970, in his famous study13, Fama gives the following definition:"A market in which prices always fully reflect the available information is called an efficient market".

A more recent definition is made by Malkiel (1992): „A capital market is called efficient if it correctly and fully reflects all relevant information in determining asset prices. Formally, the market is assumed to be efficient relative to a particular set of information, if asset prices would not be affected by revealing that information to all agents on the capital market. Furthermore, efficiency relative to a lot of information implies that it is impossible to get profits act upon that information crowds".

The first part of this latter definition is similar to Fama's classic definition, the second one involves a way to test the efficiency of capital markets: if prices do not change when a certain set of information is disclosed, then the market is efficient compared with that set of information (this test is impossible to achieve in real terms).

The third part of the definition suggests another way to measure efficiency: by measuring the profits from transactions based on a set of information, we can decide whether the efficient market hypothesis is confirmed or not. And this way is difficult to implement since the information available to the the agents in capital market as incomplete known.

One way to avoid these difficulties in testing the efficiency of capital market is to develop a classification according to the multitude of information available, so we can distinguish between three types of efficiency:

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7 Keynes, J. M., 1923, "Some Aspects of Commodities Markets", *Manchester Guardian Commercial*, March 29, reprinted in *The Collected Writings of John Maynard Keynes*, *Volume XII*, London: Macmillan, 1983.

8 Olivier, M., 1926, "Les Nombres indices de la variation des prix", Paris doctoral dissertation.

9 Larson, A. B., 1960, "Measurement of a Random Process in Futures Prices", *Food Research Institute Studies*, Vol. **1**, No. 3, pp. 313-24.

10 Mandelbrot, B., 1963, "The Variation of Certain Speculative Prices", *The Journal of Business*, Volume **36**, Issue 4, October, pp. 394-419.

11 Fama, E. F., 1963, "Mandelbrot and the Stable Paretian Hypothesis", *The Journal of Business*, Volume **36**, Issue 4, October, pp. 420-429.

12 *Mean absolute deviation* (MAD) for a set of values $x_1,\ldots,x_n$ is defined as: $MAD=\sum_{i=1}^{n}\frac{|x_i-\bar{x}|}{n}$

13 Fama E. F., 1970. "Efficient Capital Markets: A Review of Theory and Empirical Work", *Journal of Finance*, **25**(2), pp. 383–417.

- **weak form efficiency—**the set of information includes only the transaction history (information on prices or financial return on assets);
- **semi-strong form efficiency—**the set of information includes, besides the transaction history, all public information known by all participants in the transaction;
- **strong form efficiency—**sets of information including all information known to any of the capital market actors (including private information).

One way to test the efficiency of capital market is to study the behavior of stock returns: if these are unpredictable it is an indication that the market is efficient.

A reverse argument is offered by so-called *law of iterated averages*. Allowing in this sense two sets of information I_t and J_t, so that $I_t \subset J_t$, namely the second set is superior in information to the first one.

The law of iterated averages says this: if X is a random variable, then $\mathbf{E}[X \mid I_t] = \mathbf{E}[\mathbf{E}[X \mid J_t] \mid I_t]$.

Interpretation of this law is the following: prediction based on the information contained in the set I_t is identical to the prediction that we get it if we have the additional information contained in the set J_t.

Applying this law in equity markets leads to an interesting conclusion: if a market is efficient, fluctuations in financial asset prices are not predictable.

Indeed, suppose that at some point we have the set of information I_t, information that is completely and correctly reflected in the price P_t (formalizing, this means that there is a random variable V, so that $P_t = \mathbf{E}[V \mid I_t]$).

Similarly, the price of the next moment *t +1*, is determined by a set of information $I_{t+1} \supset I_t : P_{t+1} = \mathbf{E}[V \mid I_{t+1}]$.

Then the expected value of price change between the two moments of time is:

$$\mathbf{E}[P_{t+1} - P_t \mid I_t] = \mathbf{E}\{[\mathbf{E}[V \mid I_{t+1}] - \mathbf{E}[V \mid I_t]] \mid I_t\}$$
$$= \mathbf{E}\{[\mathbf{E}[V \mid I_{t+1}] \mid I_t\} - \mathbf{E}[V \mid I_t] = 0 .$$

In conclusion, price change cannot be predicted, based on the information contained in the set I_t.

From a theoretically view point, if the issue of the capital market efficiency is well devised, we question in what way it can be extented to achieve a practically efficient market hypothesis testing. A useful approach to achieve this is the concept of relative efficiency, i.e. testing the efficiency of a market in relation to another market.

The possibility to model the behaviour of financial assets in order to achive predictions of their future returns is a concern of researchers in this field. In the following we consider the problem of predictability of financial asset price fluctuations, considering that they are influenced by past values.

3.1.RW1 hypothesis: independent and identically distributed increments (i.i.d)

The most natural expression of random walk hypothesis is that the price of financial assets is a stohastic process with internal dependence, with the following form:

$$P_t = \mu + P_{t-1} + \varepsilon_t , \qquad (3.1)$$

where $(\varepsilon_t)_t \sim \mathbf{WN}(0,\sigma^2)$ is a white noise, i.e. a series of random variables independent, identically distributed:

$$E[\varepsilon_t] = 0, \forall t$$
$$Var[\varepsilon_t] = \sigma^2, \forall t$$
$$\varepsilon_t, \varepsilon_{t+k} \textit{ independents for every } k$$

Moreover, if the last condition is satisfied, then we have.

$$\mathbf{Cov}\left[\varepsilon_t, \varepsilon_{t+k}\right] = 0 \text{ şi } \mathbf{Cov}\left[\varepsilon_t^2, \varepsilon_{t+k}^{\ 2}\right] = 0, \forall k \neq 0 .$$

In equation (3.1), P_t, P_{t-1} are the price value at two consecutive moments, and μ is the expected price change, so-called *drift*.

Independence of innovations $(\varepsilon_t)_t$ implies that random walk is also a fair game, but in a sense stronger than the Martingale hypothesis: increments are not only non-correlated, but also independent, hence results that any linear combinations thereof are non-correlated.

The functional form of the RW1 model induces non-stationarity conditions of the process

$$(P_t)_t : \begin{cases} \mathbf{E}[P_t \mid P_0] = P_0 + \mu t \\ \mathbf{Var}[P_t \mid P_0] = \sigma^2 t \end{cases} .$$

The most encountered condition that is imposed to innovations $(\varepsilon_t)_t$ is the one of the normality, beside the white noise's character, a condition which induces a certain convenience in formal terms. But it appears inconsistent with the actual situation, because the normal distribution covers the entire real line, so there is a nonzero probability that an asset price is negative. One way to avoid this difficulty is to use instead of financial asset price series, time series of natural logarithms of these prices: $p_t = \log P_t$.

RW1 model becomes then a lognormal:

$$p_t = \mu + p_{t-1} + \varepsilon_t ,$$

where $(\varepsilon_t)_t \sim \mathbf{WN}(0,\sigma^2)$ (i.e. white noise) and $\varepsilon_t \sim \mathbf{N}(0,\sigma^2)$.

3.2. RW2 Hypothesis: independent increments

Although simplicity and elegance RW1 model is appealing, assuming the existence of independent identically distributed growth is just natural.

Factors that determine the evolution of financial asset prices in a market are not the same and do not have the same intensity for different periods of time. Also, economic conditions differ greatly over time, making the identical distributions assumption over the entire time horizon to be unnatural.

When the RW2 model is derived from RW1 model, renouncing to the hypothesis of the existence of a joint distribution of innovations $(\varepsilon_t)_t : P_t = \mu + P_{t-1} + \varepsilon_t$, where is a sequence of independent random variables such that

$$\begin{cases} \mathbf{E}[\varepsilon_t] = 0, \forall t \\ \mathbf{Var}[\varepsilon_t] = \sigma_t^2, \forall t \\ \mathbf{Cov}[\varepsilon_t, \varepsilon_{t+k}] = 0, \forall k \neq 0 \\ \mathbf{Cov}[\varepsilon_t^2, \varepsilon_{t+k}^2] = 0, \forall k \neq 0 \end{cases}.$$

Although the RW2 model is weaker than RW1, retains its essence: any future change in the price of financial assets is unpredictibil using past price changes.

3.3.RW3 hypothesis: non-correlated increments

Relaxing the assumptions of previous models, we obtain a more general form of the random walk hypothesis, the innovations are dependent but non-correlated.

$$P_t = \mu + P_{t-1} + \varepsilon_t$$

where $(\varepsilon_t)_t$ is a sequence of random variables such that

$$\begin{cases} \mathbf{E}[\varepsilon_t] = 0, \forall t \\ \mathbf{Var}[\varepsilon_t] = \sigma_t^2, \forall t \\ \mathbf{Cov}[\varepsilon_t, \varepsilon_{t+k}] = 0, \forall k \neq 0 \end{cases}.$$

An immediate consequence of the efficient market hypothesis in weak form is that price changes (i.e., yields) are not predictable.

One of the most commonly used statistical test to verify the hypothesis of random walks is variance ratio test. An important property of all random walk hypothesis is that the variable residual variance to be a linear function of time.

Considering the RW1 model $r_t = \mu + \varepsilon_t$, since yields r_t are independent and follows the same distribution, we have that $\mathbf{Var}[r_t + r_{t-1}] = 2\mathbf{Var}[r_t]$. Therefore, we can determine whether the random walk hypothesis is plausible verifying report variances: $VR(2) = \frac{\mathbf{Var}[r_t + r_{t-1}]}{2\mathbf{Var}[r_t]}$. If RW1 hypothesis is true, then this report should be substantially equal to one.

Variances ratio can be written according to first-order autocorrelation coefficient, if it is assuming that the return series is stationary (this is necessary to define the autocorrelation function):

$$VR(2) = \frac{\mathbf{Var}[r_t + r_{t-1}]}{2\mathbf{Var}[r_t]} = \frac{2\mathbf{Var}[r_t] + 2\mathbf{Cov}[r_t, r_{t-1}]}{2\mathbf{Var}[r_t]} = 1 + 2\rho(1) \cdot$$

If RW1 is met, then a first-order autocorrelation coefficient is zero, so $VR(2) = 1$.If the series has positive autocorrelation of first-order, then $VR(2) > 1$, and if the series has negative autocorrelation of first-order, then $VR(2) < 1$.

For lags bigger than 1, variances ratio is a linear combination of coefficients's autocorrelation:

$$VR(q) = \frac{\mathbf{Var}[r_t + r_{t-1} + ... + r_{t-q+1}]}{q\mathbf{Var}[r_t]} = 1 + 2\sum_{k=1}^{q-1}\left(1 - \frac{k}{q}\right)\rho(k) \cdot$$

To infer the distribution of $VR(2)$, we assume that profitability r_t follows a pattern RW1: $H_0 : r_t = \mu + \varepsilon_t$, where $(\varepsilon_t)_t$ is a sequence of independent random variables indentically distributed $\varepsilon_t \sim \mathbf{N}(0, \sigma^2)$.Assuming that we work with a sample of *2n+1* observations over time $p_0, ..., p_{2n}$, we will consider the following estimators for the parameters's distribution, μ and σ^2:

- $\hat{\mu} = \frac{1}{2n}\sum_{k=1}^{2n}(p_k - p_{k-1}) = \frac{1}{2n}(p_{2n} - p_0)$
- $\hat{\sigma}_a^2 = \frac{1}{2n}\sum_{k=1}^{2n}(p_k - p_{k-1} - \hat{\mu})^2$
- $\hat{\sigma}_b^2 = \frac{1}{2n}\sum_{k=1}^{n}(p_{2k} - p_{2k-2} - 2\hat{\mu})^2$.

Estimators $\hat{\mu}$ and $\hat{\sigma}_a^2$ are exactly the estimators of maximum verosimility of the two parameters, and $\hat{\sigma}_b^2$ is an estimator constructed so that is taking into account the random behavior of time series $(p_t)_t$; variance is a linear function of time, so σ^2 can be estimated by half of the variance even-terms of the series.

Then in contions of RW1 hypothesis , we can inferre the asymptotic distribution of the variances ratio $\hat{VR}(2) = \frac{\hat{\sigma}_b^2}{\hat{\sigma}_a^2} : \sqrt{2n}(\hat{VR}(2) - 1) \to \mathbf{N}(0,2)$ [14].

We reject the random walk hypothesis at significance level α if the value of statistics $z = \frac{\sqrt{2n}(\hat{VR}(2) - 1)}{\sqrt{2}}$ is outside the interval $[-z_{\alpha/2}, z_{\alpha/2}]$.

Variances ratio test can be easily extended to the case of several time periods. If the initial sample consists of *nq+1* observations, **{** $p_0, ..., p_{nq}$ **}**, we have:

- $\hat{\mu} = \frac{1}{nq}\sum_{k=1}^{nq}(p_k - p_{k-1}) = \frac{1}{nq}(p_{nq} - p_0)$
- $\hat{\sigma}_a^2 = \frac{1}{nq}\sum_{k=1}^{nq}(p_k - p_{k-1} - \hat{\mu})^2$
- $\hat{\sigma}_b^2(q) = \frac{1}{nq}\sum_{k=1}^{nq}(p_{kq} - p_{kq-q} - q\hat{\mu})^2$
- $\hat{VR}(q) = \frac{\hat{\sigma}_b^2(q)}{\hat{\sigma}_a^2}$.

In RW1 hypothesis's conditions, we have:

[14] Haussman, J. 1978, "Specification Tests in Econometrics", *Econometrica*, Vol. **46**.

$\sqrt{nq}(VR(q)-1) \rightarrow \mathbf{N}(0,2(q-1))$.

We can refine the asymptotic distribution of the variances ratio, building better estimators for the parameters from above.A better estimator for the model's dispersion can be obtained using returns for q periods:

$$\hat{\sigma}_c^2(q) = \frac{1}{nq^2}\sum_{k=q}^{nq}(p_k - p_{k-q} - q\hat{\mu})^2 \cdot$$

Also, we will correct dispersions $\hat{\sigma}_a^2$ and $\hat{\sigma}_b^2$:

- $\bar{\sigma}_a^2 = \frac{1}{nq-1}\sum_{k=1}^{nq}(p_k - p_{k-1} - \hat{\mu})^2$

- $\bar{\sigma}_b^2(q) = \frac{1}{m}\sum_{k=1}^{nq}(p_{kq} - p_{kq-q} - q\hat{\mu})^2$,

where $m = q(nq - q + 1)\left(1 - \frac{1}{n}\right)$.

The new ratio of the variances will be: $\overline{VR}(q) = \frac{\bar{\sigma}_c^2(q)}{\bar{\sigma}_a^2}$.

Then we define a new statistics by which we test the hypothesis of random walk:

$$\psi(q) = \sqrt{nq}(\overline{VR}(q)-1)\left(\frac{2(2q-1)(q-1)}{3q}\right)^{-1/2} \rightarrow \mathbf{N}(0,1) \cdot$$

This test can be used to verify the RW1 hypothesis, assuming homoscedasticity and returns independence.

A variant of this test, for the RW3 hypothesis, in which we presume the heteroscedasticity hypothesis, is presented below.

The two versions of the ratio-variance test follows the methodology used by Lo and MacKinlay (1988) and Campbell, Lo, and MacKinlay (1997).

To test RW3, assuming innovation's non-correlation and heteroscedasticity, one can use the following statistics:

$$\psi^*(q) = \frac{\sqrt{nq}(VR(q)-1)}{\sqrt{\hat{\theta}(q)}} \approx N(0,1).$$

- Variance ratio is computed using its asymptotic expression: $VR(q) \rightarrow 1 + 2\sum_{k=1}^{q-1}\left(1 - \frac{k}{q}\right)\hat{\rho}(k)$, where $\hat{\rho}(k)$ is the coefficient's estimator of autocrelation of order k for r_t;

- $\hat{\theta}(q)$ is an heteroscedasticity-consistent estimator of $\theta(q)$, the asymptotic dispersion of VR(q), calculated such as:

$$\hat{\theta}(q) = 4\sum_{k=1}^{q-1}\left(1 - \frac{k}{q}\right)^2 \hat{\delta}_k ;$$

- $\hat{\delta}_k$ is a heteroscedasticity-consistent estimator of δ_k, the asymptotic dispersion of $\rho(k)$, r_t's autocorrelation coefficient of order k:

$$\hat{\delta}_k = \frac{nq\sum_{j=k+1}^{nq}(p_j - p_{j-1} - \hat{\mu})^2(p_{j-k} - p_{j-k-1} - \hat{\mu})^2}{\left[\sum_{j=1}^{nq}(p_j - p_{j-1} - \hat{\mu})^2\right]^2} = .$$

$$= \frac{nq\sum_{j=k+1}^{nq}(r_j - \hat{\mu})^2(r_{j-k} - \hat{\mu})^2}{\left[\sum_{j=1}^{nq}(r_j - \hat{\mu})^2\right]^2}$$

Typically, to test one of the two forms of the random walk hypothesis, we compute the variance ratios *VR(q)* for *m* periods, $\{q_1, q_2, ..., q_m\}$, then determine appropriate statistics $\psi(q)$ or $\psi^*(q)$ and finally compare the statistics with a critical value of normal or Student distribution, $z_{\alpha/2}$ or $t_{\alpha/2;nq}$.

4. APPLICATION FOR BET INDEX

To check the assumptions from above, we used daily data values of BET, the index of the Bucharest Stock Exchange. The time period studied is 19th September 1997 to 15th June 2010(3164 observations).We used logreturn, defined as $r_t = \ln P_t - \ln P_{t-1}$, where P_t is the index value at time t.

4.1 Normal distribution assumption

To check the hypothesis of normal distribution for daily return of BET index, we applied the battery of tests for normal distribution available in SAS 9.2: Kolmogorov-Smirnov test, Anderson-Darling test and Cramer-von Mises test. In all three cases, the normal distribution hypothesis was rejected with a probability of at least 99%.

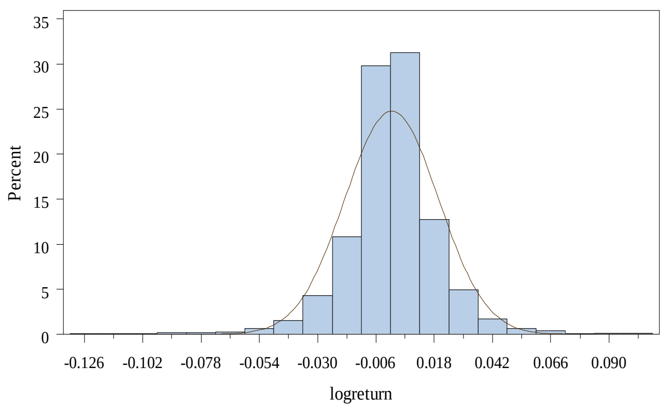


Fig. 2. Histogram of logreturn for BET

Table 1. Parameters of normal distribution

Parameter	Estimate
Mean	0.000506
Std Dev	0.019321

Table 2. Goodness-of-Fit Tests for Normal Distribution

Test	**Statistic**		**p Value**	
Kolmogorov-Smirnov	**D**	0.085	**Pr >D**	<0.010
Cramer-von Mises	**W-Sq**	8.062	**Pr>W-Sq**	<0.005
Anderson-Darling	**A-Sq**	45.963	**Pr>A-Sq**	<0.005

Moreover, the normal distribution cannot explain the presence of large deviations in stock price evolution.

The table below shows the probability that returns are lower than a certain value, computed form the estimated normal distribution and from the real data

Table 3. Distribution of extrem returns

	$\Pr(r_t > c)$	
Cut point(c)	Real data	Normal distribution
-0.05	0.013906	0.0044738
-0.1	0.001264	9.863E-08
-0.11	0.000948	5.343E-09
-0.13	0.000316	7.161E-12

A strong research direction studied in later years, although it has its origins in the works of Mandelbrot in the ‚60s, is the use of stable distributions (Pareto-Levy) for modeling stocks's returns.

As noted, the return's distribution has tails of much higher return than would be expected under normal distribution, and stable distributions resolves the problem of such extreme events. Stable distributions have a remarquable property: they allow for skewness and heavy tails and more, any linear combination of stable independent variables is also stable. In other words, the shape of distribution is preserved under linear transformation.

In general, the stable distributions do not present an explicit form of probability density function, being only the known characteristic function. Normal distribution is a special case of stable distribution: any linear combination of dependent Gaussian is also Gaussian.

In literature there are several parametrizations of stable distributions. We chosed for this paper the parametrisation S0, in Nolan (2001)'s variant.

Thus, a variable X follow a stable distribution $S(\alpha, \beta, \gamma, \delta; 0)$ if its characteristic function has the form:

$$\phi(t) = \mathrm{E}[e^{itX}] = \begin{cases} \exp(-\gamma^{\alpha}|t|^{\alpha}[1 + i\beta \tan(\frac{\pi\alpha}{2}) sign(t)(|\gamma t|^{1-\alpha} - 1)] + i\delta t, \alpha \neq 1 \\ \exp(-\gamma|t|[1 + i\beta t \frac{2}{\pi} sign(t)(\ln(\gamma|t|)] + i\delta t, \alpha = 1 \end{cases}.$$

In the above notation $\alpha \in (0.2]$ is the characteristic parmeter(for normal distribution $\alpha = 2$), $\beta \in [-1,1]$ is the skewness parameter, $\gamma \in (0,\infty)$ is the scale parameter and $\delta \in \mathbf{R}$ is the location parameter.For our daily return series of BET index, the following results have been obtained, using the software STABLE (Nolan, 2001).

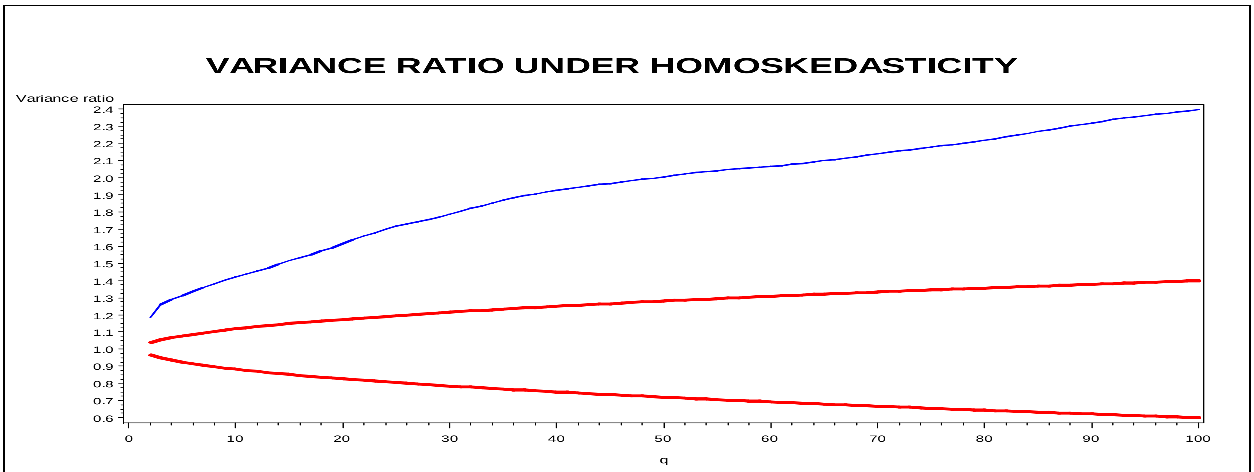
Table 4. Parameters of the stable distribution

Parameter	**Estimate**	**Lower 95%**	**Upper 95%**
α	1.476234	1.421034	1.531434
β	-0.01872	-0.138416	0.100984
γ	0.009246	0.0088915	0.0096011
δ	0.000551	-0.00005	0.0011552

Maximum likelihood estimators of stable distribution under S0 parametrization shows that we can reject the normal distribution hypothesis, since the characteristic parameter α is significantly lower than 2, the value of the gaussian distribution.

4.2. Random walk hypothesis

Based on the methodolgy descrided above, we have computed variance ratios for daily returns. Also a confidence interval with 95% probability was computed, using homoskedasticity or heteroskedascicity assumption for model innovations.



VARIANCE RATIO UNDER HETEROSKEDASTICITY

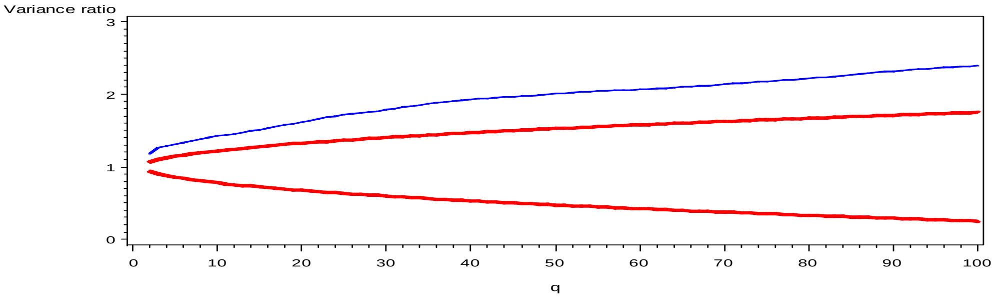


Fig. 3. Confidence intervals for VR(q)

In the above graphs, the red lines represent the limits of 95% confidence interval, while the blue line represents the values of variance ratio.Based on the Variance Ratio Test, we can reject the random walk hypothesis for daily series of BET.

5. CONCLUSIONS

Altough the normal distribution has been widely used for a lot of applications in the financial world we still need appropriate distribution in order to take account for large variability and heavy tails. Stable distributions are a good approach for these problems even they are not easy to define analytically and also easy to estimate.

Also the Efficient Market Hypothesis (and consequently the Random Walk Hypothesis) needs to be reconsidered, since they cannot explain large fluctuations in stock price and stock market crisis.

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